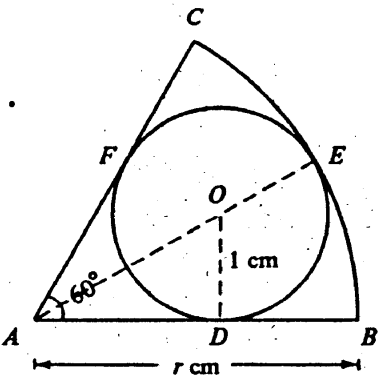


# RESTRICTED 内部文件

87 MATHS (SYLL A/B)

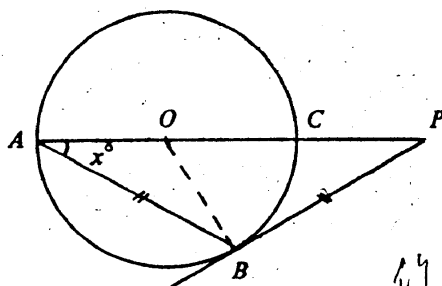
P.1

SOLUTION	MARKS	REMARKS
<p>1. (a) <math>x^2 - 2x + 1 = (x - 1)^2</math></p> <p>(b) <math>x^2 - 2x + 1 - 4y^2 = (x - 1)^2 - 4y^2</math></p> <p style="margin-left: 150px;"><math>= (x - 1 - 2y)(x - 1 + 2y) \dots</math></p> <p style="margin-left: 150px;"><math>= (x - 2y - 1)(x + 2y - 1)</math></p> <p style="margin-left: 150px;"><math>(x-1-2y)(x-1+2y)</math></p>	<p>2A</p> <p>1M</p> <p>1M+1A</p> <p style="text-align: center;"><u>5</u></p>	<p>or <math>(x-1)(x-1)</math></p> <p>for <math>( )^2 - 4y^2</math></p> <p>1M for diff. of 2 sq's. No marks for <math>x^2 - 4y^2 = (x-2y)(x+2y)</math></p>
<p>2. Let <math>f(x) = 2x^3 + ax^2 + bx - 2</math></p> <p>Putting <math>x = 2</math>,</p> <p style="margin-left: 100px;"><math>f(2) = 4a + 2b + 14</math></p> <p>As <math>x - 2</math> divides <math>f(x)</math>, <math>4a + 2b + 14 = 0</math>.</p> <p>Similarly</p> <p style="margin-left: 100px;"><math>f(-1) = a - b - 4</math></p> <p style="margin-left: 100px;"><math>= 0</math></p> <p>Solving the equations, <math>6a + 6 = 0</math></p> <p><math>a = -1</math>, <math>b = -5</math></p>	<p>1A</p> <p>1M</p> <p>1A</p> <p style="text-align: center;"><u>1A+1A</u> <u>5</u></p>	<p>for <math>f(2) = 0</math></p> <p>or <math>f(-1) = 0</math></p>
<p>(Syll A)</p> <p>3. (a) <math>\sqrt{\frac{3^{5k+2}}{27^k}} = \sqrt{\frac{3^{5k+2}}{(3^3)^k}}</math></p> <p style="margin-left: 100px;"><math>= 3^{k+1} \dots \dots \dots</math></p> <p>(b) <math>\frac{\log a^3 b^2 - \log ab^2}{\log \sqrt{a}} = \frac{\log \frac{a^3 b^2}{ab^2}}{\log \sqrt{a}} \dots \dots \dots</math></p> <p style="margin-left: 150px;"><math>= \frac{\log a^2}{\log \sqrt{a}}</math></p> <p style="margin-left: 150px;"><math>= \frac{2 \log a}{\frac{1}{2} \log a} \dots \dots \dots</math></p> <p style="margin-left: 150px;"><math>= 4</math></p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p style="text-align: center;"><u>1A</u> <u>5</u></p>	<p>or</p> <p><math>= \frac{\log a^3 + \log b^2 - \log a - \log b^2}{\log \sqrt{a}}</math></p> <p style="text-align: right;">1A</p> <p><math>= \frac{3 \log a - \log a}{\frac{1}{2} \log a}</math></p> <p style="text-align: right;">1A</p>
<p>(Syll B)</p> <p>3. <math>3^{2x} + 3^x - 2 = 0</math></p> <p style="margin-left: 50px;"><math>(3^x)^2 + 3^x - 2 = 0 \dots \dots \dots</math></p> <p style="margin-left: 50px;"><math>(3^x - 1)(3^x + 2) = 0</math></p> <p style="margin-left: 50px;"><math>3^x = 1</math> or <math>3^x = -2</math></p> <p>(Rejecting <math>3^x = -2</math>)</p> <p style="margin-left: 50px;"><math>x = 0</math></p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p style="text-align: center;"><u>1A</u> <u>5</u></p>	<p><math>(3^x)^2</math></p> <p>)</p> <p>) Accept <math>3^x = 1</math></p> <p>)</p>

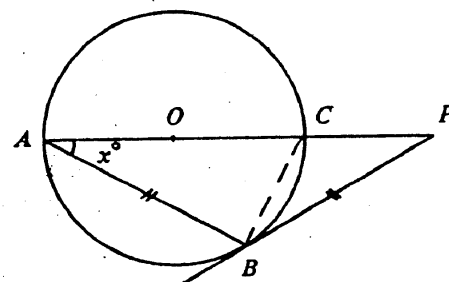
SOLUTION	MARKS	REMARKS
<p>4. <math>\sin^2 \theta = \frac{3}{2} \cos \theta</math></p> <p><math>1 - \cos^2 \theta = \frac{3}{2} \cos \theta</math></p> <p><math>2\cos^2 \theta + 3\cos \theta - 2 = 0</math></p> <p><math>(2\cos \theta - 1)(\cos \theta + 2) = 0</math></p> <p><math>2\cos \theta = 1</math> or <math>\cos \theta = -2</math></p> <p>Rejecting <math>\cos \theta = -2</math>, we have</p> <p><math>\cos \theta = \frac{1}{2}</math></p> <p><math>\theta = 60^\circ</math> or <math>300^\circ</math> (or <math>\frac{\pi}{3}</math>, <math>\frac{5\pi}{3}</math>)</p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A+1A</p> <p><u>6</u></p>	<p>)</p> <p>) Accept <math>2\cos \theta = 1</math></p> <p>)</p> <p>-1 for each extraneous solution</p>
<p>5. <math>kx^2 - 4x + 2k = 0</math></p> <p>(4) <math>\alpha + \beta = \frac{4}{k}</math></p> <p><math>\alpha\beta = 2</math></p> <p>(a) <math>\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta</math></p> <p><math>= \left(\frac{4}{k}\right)^2 - (2)(2)</math></p> <p><math>= \frac{16}{k^2} - 4</math></p> <p>(b) <math>\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}</math></p> <p><math>= \left(\frac{16}{k^2} - 4\right) \cdot \frac{1}{2}</math></p> <p><math>= \frac{8}{k^2} - 2</math></p>	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p> <p><u>1A</u></p> <p><u>6</u></p>	<p>or <math>\frac{16-4k^2}{k^2}</math></p> <p>or equivalent.</p>
<p>6. By symmetry, <math>\angle BAE = 30^\circ</math></p> <p>AS <math>OD \perp AB</math>,</p> <p><math>\sin 30^\circ = \frac{1}{AO}</math></p> <p><math>\therefore AO = 2</math></p> <p><math>AE = AO + OE</math></p> <p><math>= 2 + 1</math></p> <p><math>= 3</math></p> <p><math>AB = AE</math></p> <p><math>\therefore r = 3</math></p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p><u>1A</u></p> <p><u>6</u></p>	

SOLUTION	MARKS	REMARKS
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7.



作線 1分



Join OB.

As OA and OB are radii of the same circle,

$$\angle OBA = \angle PAB = x^\circ \dots\dots\dots$$

Since PB is a tangent,

$$\angle OBP = 90^\circ$$

Given that BA = BP

$$\angle BPA = \angle PAB = x^\circ \dots\dots\dots$$

$$x + x + x + 90 = 180$$

共有  $\begin{cases} 3x = 90 \\ x = 30 \end{cases}$  得2分

這解法有三  
第一步 二x 祇  
第二步 二x 祇  
第三步 二x 祇

Alternatively:

1A Join BC.

As PB is a tangent,  
 $\angle CBP = \angle PAB = x^\circ$ .

1A

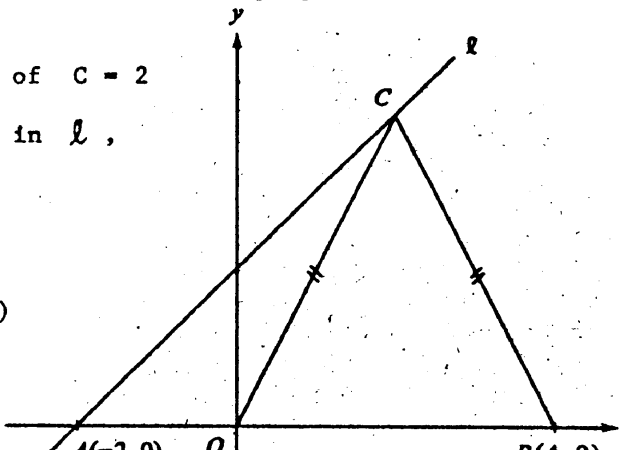
Since AC is a diameter,  $\angle ABC = 90^\circ$  etc.

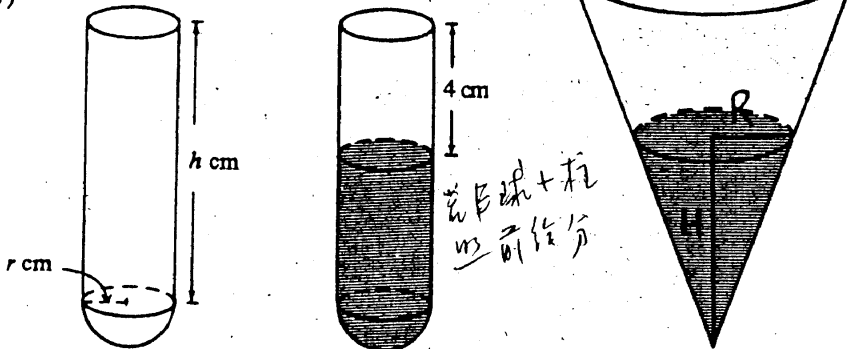
1A

1A

1A

1A  
6

SOLUTION	MARKS	REMARKS
<p>8. (a) Equation of <math>l</math> is <math>y - 0 = (1)[x - (-2)]</math>  <math>\textcircled{2}</math> i.e. <math>y = x + 2</math> (or <math>x - y + 2 = 0</math>)</p>	1A <u>1A</u> <u>2</u>	
<p>(b) As <math>CO = CB</math>, <math>C</math> lies on the perpendicular bisector of <math>OB</math>.  <math>\textcircled{3}</math> x-coordinate of <math>C = 2</math>          Substituting in <math>l</math>,  <math>y = 2 + 2</math>  <math>= 4</math>  <math>\therefore C = (2, 4)</math></p> 	1M  1A  1A  <u>3</u>	<p><u>Alternatively:</u>          Let <math>C = (x, y)</math>  <math>\sqrt{x^2 + y^2} = \sqrt{(x-4)^2 + y^2}</math> 1M  <math>8x = 16</math>  <math>x = 2</math> ..... 1A</p>
<p>(c) Let the equation of the circle be  <math>\textcircled{4}</math> <math>x^2 + y^2 + ax + by + c = 0</math>.          Substituting <math>(x, y) = (0, 0)</math> or <math>(4, 0)</math> or <math>(2, 4)</math>,  <math>c = 0</math>  <math>16 + 4a = 0</math>  <math>4 + 16 + 2a + 4b = 0</math>  <math>a = -4</math>  <math>b = -3</math>  <math>\therefore</math> the equation of the circle is  <math>x^2 + y^2 - 4x - 3y = 0</math>.</p> <p>Method  <math>(x-h)^2 + (y-k)^2 = r^2</math>  <math>h = 1, k = 1.5</math> — 1M.          Centre <math>(1, 1.5)</math>          Radius <math>1.5</math>          eq. of circle</p>	<p>3-7? 1M  <math>\textcircled{1M}</math>          1A          1A          1A          1A  <u>4</u></p>	<p><u>Alternatively:</u>          The centre of the circle lies on the perpendicular bisector of <math>OB</math> (or <math>OC</math>, <math>BC</math>) 1M          Let it be <math>(2, y)</math>  <math>(2-0)^2 + (y-0)^2 = (2-4)^2 + (y-4)^2</math>  <math>y = 3/2</math>          The centre is <math>(2, \frac{3}{2})</math> 1A          Radius of circle  <math>= \sqrt{4 + \frac{9}{4}} = \frac{5}{2}</math> ..... 1A  <math>\therefore</math> eqn. of circle is  <math>(x-2)^2 + (y-\frac{3}{2})^2 = \frac{25}{4}</math> 1A          or  <math>x^2 + y^2 - 4x - 3y = 0</math></p>
<p>(d) Substituting <math>y = x + 2</math> in the equation of the circle,  <math>\textcircled{3}</math> <math>x^2 + (x+2)^2 - 4x - 3(x+2) = 0</math>  <math>2x^2 - 3x - 2 = 0</math>  <math>(2x+1)(x-2) = 0</math>  <math>x = 2</math> or <math>-\frac{1}{2}</math>          Putting <math>x = -\frac{1}{2}</math>  <math>y = \frac{3}{2}</math>  <math>\therefore D = (-\frac{1}{2}, \frac{3}{2})</math></p>	<p>1M      1A  1A  <u>3</u></p>	

SOLUTION	MARKS	REMARKS
<p>9. (a) (i) Capacity of hemispherical part</p> <p>(3) + (3)</p> $= \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)\pi r^3$ <p>若能写出有 <math>r^3</math> 之方根 则可得 1M.</p> $= \left(\frac{1}{6}\right)(108\pi)$ $r^3 = 27$ $r = 3 \dots\dots\dots$ <p>Capacity of cylindrical part</p> $= \pi r^2 h$ <p>若 <math>r</math> 有错, 仍给此 1M</p> $= 9\pi h \dots\dots\dots$ $9\pi h = \left(\frac{5}{6}\right)(108\pi)$ $h = 10$ <p>(ii) Volume of space = <math>\pi(3^2)(4)</math></p> <p>(3)</p> <p>Volume of water = <math>108\pi - (\pi)(3^2)(4)</math></p> <p>108π ~ 1M 若 <math>r</math> 有错 1M 若不以 <math>\pi</math> 乘 不给</p> $= 72\pi \text{ cm}^3$ <p>1M+1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>Alternatively: Volume  <math display="block">= \pi(3)^2(10-4) + \frac{108\pi}{6}</math> <math display="block">\dots\dots\dots 1M+1M</math> <math display="block">= 72\pi \text{ cm}^3 \dots\dots\dots 1A</math></p> <p>(b)</p> <p>(3)</p>  <p>Let radius and depth of water be R and H.</p> $\frac{1}{3}\pi R^2 H = (72\pi)$ <p>若能写出此 1M 分</p> $R^2 H = 216$ <p>Capacity of vessel = <math>\frac{1}{3}\pi(2R)^2(2H)</math></p> $= \frac{8}{3}\pi R^2 H$ $= \frac{8}{3}\pi(216)$ $= 576\pi \text{ cm}^3 \dots\dots\dots$ <p>1M</p> <p>1M</p> <p>1A</p> <p>3</p> <p>Alternatively: Since height of vessel = 2 X height of water Capacity of vessel = <math>2^3 \times 72\pi</math>  <math display="block">= 576\pi \text{ cm}^3 \dots\dots\dots</math></p> <p>2M</p> <p>1A</p> <p>3</p> <p>-1 if unit not given</p>		

## SOLUTION

## MARKS

## REMARKS

10. (a) Since the triangle is equilateral,  $\angle A_1 = 60^\circ$ ,

$$T_1 = \frac{1}{2} (3)(3)(\sin 60^\circ)$$

$$= \frac{9\sqrt{3}}{4}$$

1M

1A

2

(b) (i) Since  $A_2B_1 = 2$ ,  $B_1B_2 = 1$  and  $\angle B_1 = 60^\circ$ ,

$$\angle B_1B_2A_2 = 90^\circ$$

$$\therefore A_2B_2 = \sqrt{3} \dots\dots\dots$$

1M

1A

Alternatively:  
By cosine rule,

$$(A_2B_2)^2$$

$$= 2^2 + 1^2 - 2(2)(1)\cos 60^\circ$$

$$= 3$$

$$\therefore A_2B_2 = \sqrt{3}$$

(ii)  $\triangle A_2B_2C_2$  and  $\triangle A_1B_1C_1$  are similar. The ratio of their sides is  $\sqrt{3} : 3$ .

$$\therefore T_2 = \frac{9\sqrt{3}}{4} \left( \frac{\sqrt{3}}{3} \right)^2$$

$$= \frac{3\sqrt{3}}{4} \dots\dots\dots$$

1M

1A

4

(c) (i) The common ratio =  $\frac{1}{3}$  (1M, 此以十數表之  
在加5)

1M

$$(ii) T_n = \frac{9\sqrt{3}}{4} \left( \frac{1}{3} \right)^{n-1}$$

1M

$$(iii) T_1 + T_2 + \dots + T_n = \frac{9\sqrt{3}}{4} \cdot \frac{1 - \left(\frac{1}{3}\right)^n}{1 - \frac{1}{3}}$$

1M

$$= \frac{27\sqrt{3}(1 - \frac{1}{3^n})}{8}$$

1A

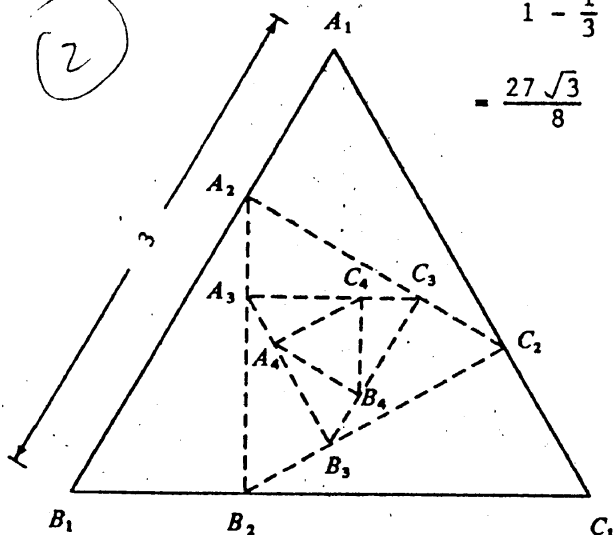
$$(iv) \text{ The sum to infinity } = \frac{\frac{9\sqrt{3}}{4}}{1 - \frac{1}{3}}$$

1M

$$= \frac{27\sqrt{3}}{8} \dots\dots\dots$$

1A

6

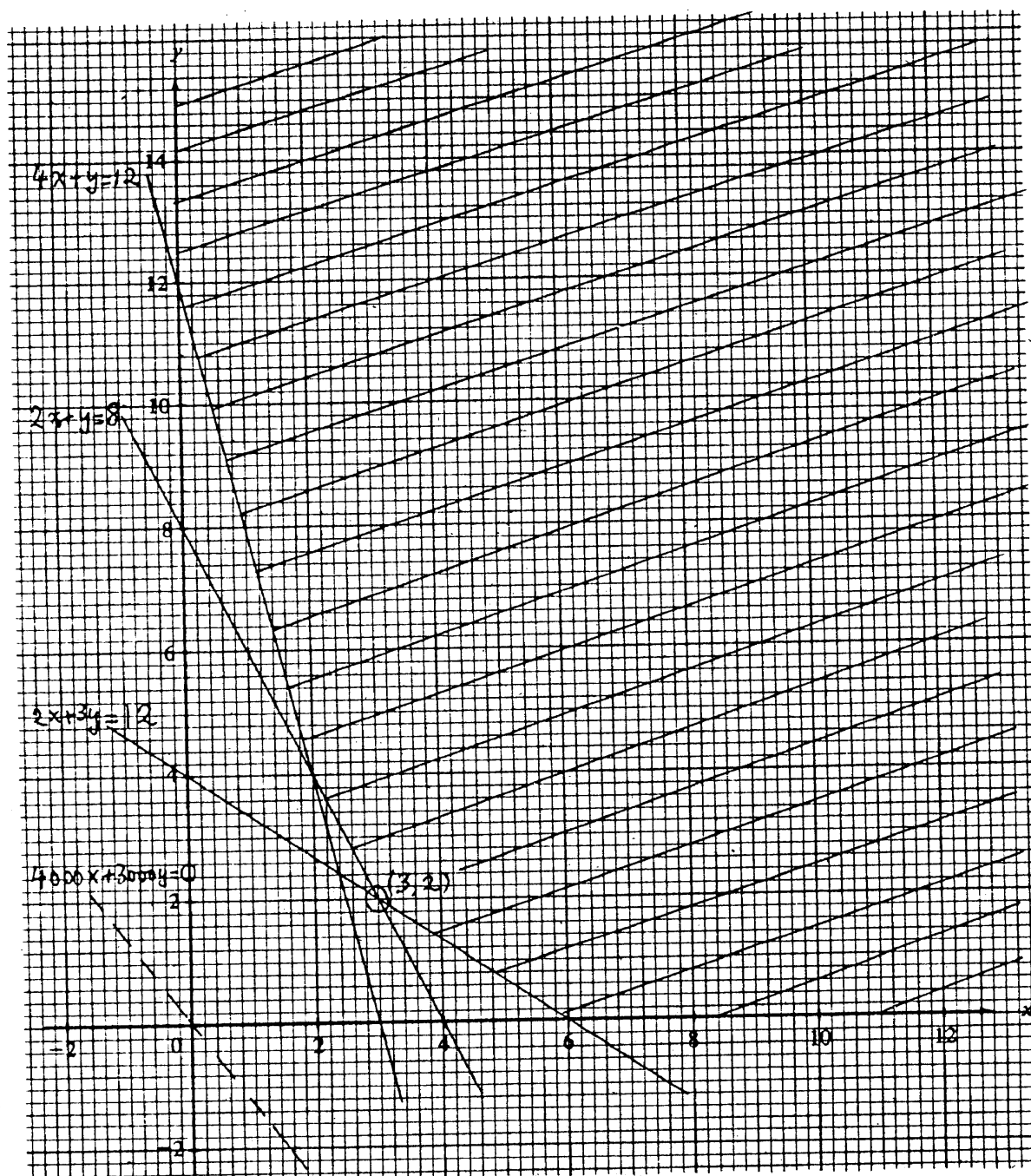




SOLUTION	MARKS	REMARKS										
<p>12. (a) Given that <math>x \geq 0</math></p> <p style="text-align: center;"><math>y \geq 0</math></p> <p><math>4000x + 6000y \geq 24\ 000</math></p> <p>Considering Products B and C ,</p> <p>(2) <math>20\ 000x + 5000y \geq 60\ 000</math></p> <p style="text-align: center;"><math>6000x + 3000y \geq 24\ 000</math></p>	<p>1A</p> <p>1A</p> <p><u>2</u></p>	<p>Withhold 1A if '=' missing</p>										
<p>(b) The constraints in (a) can be written as</p> <p style="text-align: center;"><math>x \geq 0</math></p> <p style="text-align: center;"><math>y \geq 0</math></p> <p><math>2x + 3y \geq 12</math></p> <p><math>4x + y \geq 12</math></p> <p><math>2x + y \geq 8</math></p> <p>The lines corresponding to the last 3 inequalities are shown on the graph paper.</p> <p>Shading the correct region.</p>	<p>1A+1A</p> <p>3A</p> <p><u>6</u></p>	<p>±1 unit at x,y axes</p> <p>-1 if shading not complete.</p> <p>-2 if only arrows used</p>										
<p>(c) Cost of materials used = <math>4000x + 3000y</math> (dollars)</p> <p>Drawing the line <math>4000x + 3000y = 0</math> (or equivalent)</p> <p style="text-align: center;">(要 slope 要正確) 不需要太準確</p> <p>The cost is least when <math>x = 3, y = 2</math></p> <p>and the least cost is 18 000 (dollars)</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p><u>4</u></p>	<p>Candidates may also test all vertices of given region.</p> <p>Awarded only if region correct</p> <table><tr><th>Point</th><th>Cost</th></tr><tr><td>(6,0)</td><td>24 000</td></tr><tr><td>(3,2)</td><td>18 000</td></tr><tr><td>(2,4)</td><td>20 000</td></tr><tr><td>(0,12)</td><td>36 000</td></tr></table>	Point	Cost	(6,0)	24 000	(3,2)	18 000	(2,4)	20 000	(0,12)	36 000
Point	Cost											
(6,0)	24 000											
(3,2)	18 000											
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(0,12)	36 000											

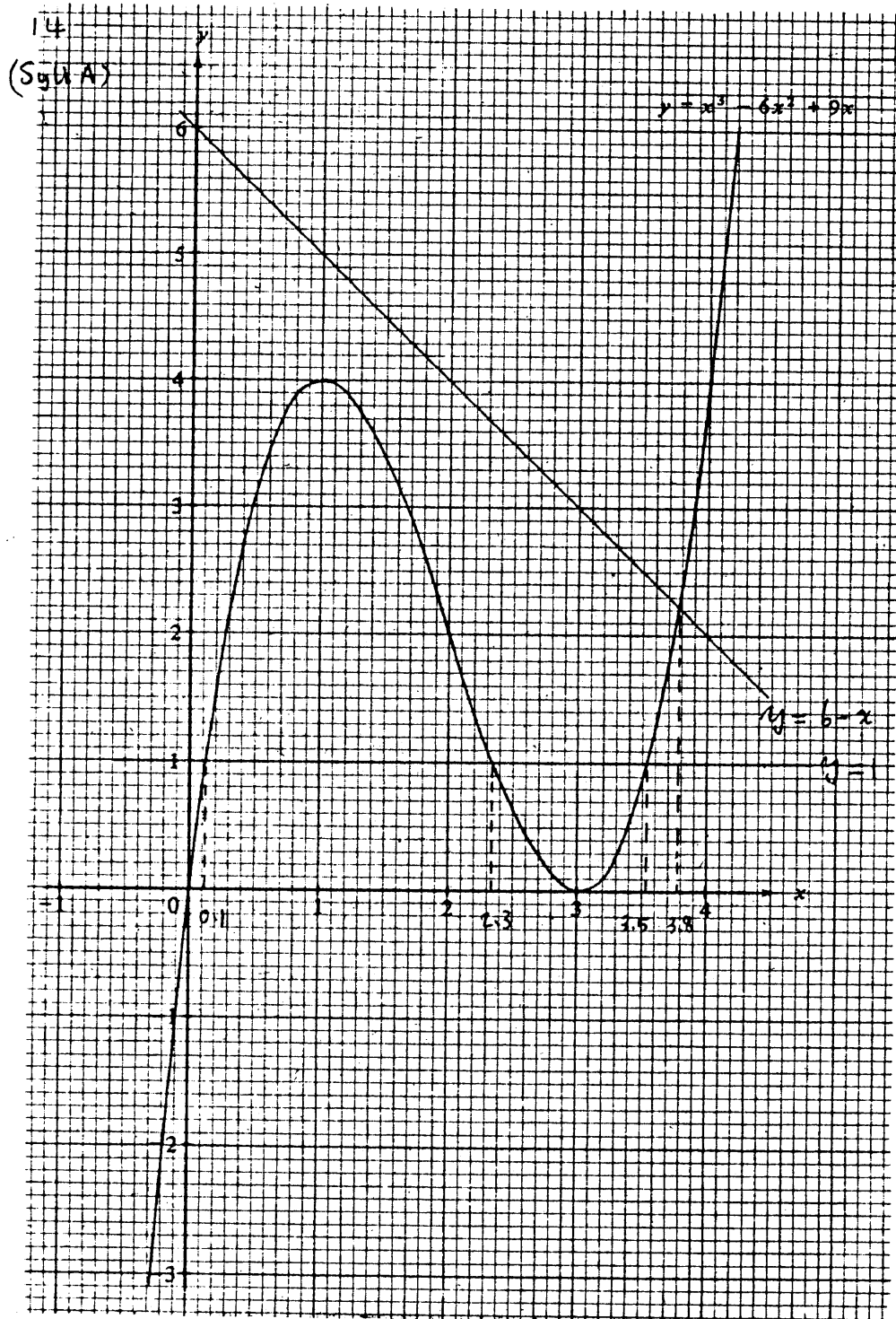


12.



SOLUTION	MARKS	REMARKS
<p>13. (a) The probability that the black ball is not drawn = <math>\frac{5}{6}</math> (or <math>1 - \frac{1}{6} = \frac{5}{6}</math>)</p> <p>(2)</p>	2A	Any value roundable to 0.83 P.P. if only answer is given. However, accept $P = 5/6$ .
	<u>2</u>	
<p>(b) The probability that the black ball is drawn from P to Q in the 1st draw = <math>\frac{1}{6}</math></p> <p>(4)</p> <p>After that, the probability that the black ball is not drawn from Q to R in the 2nd draw = <math>\frac{4}{5}</math></p> <p><math>\therefore</math> the probability that the black ball is in Q</p> <p><math>= \frac{1}{6} \times \frac{4}{5}</math></p> <p><math>= \frac{2}{15}</math> ( = <math>\frac{4}{30}</math> )</p> <p><math>\frac{1}{6} + \frac{4}{5}</math> 1+1</p> <p><math>\frac{1}{6} \times \frac{4}{5} \times \dots</math> 1+1</p>	1A  1A	
	<u>2A</u> <u>4</u>	
<p>(c) The probability that the black ball is drawn from Q to R = <math>\frac{1}{5}</math> .....</p> <p>(3)</p> <p><math>\therefore</math> the probability that the black ball is in R</p> <p><math>= \frac{1}{6} \times \frac{1}{5}</math> .....</p> <p><math>= \frac{1}{30}</math> = 0.03 <math>\frac{1}{6}</math> 1A <math>\frac{1}{5}</math> 1A</p>	1A  1A	Alternatively: $1 - \frac{5}{6} - \frac{2}{15}$ (IM) 2M
	<u>1A</u> <u>3</u>	
<p>(d) The probability that a white ball is drawn from P to Q in the 1st draw = <math>\frac{3}{6}</math> ( = <math>\frac{1}{2}</math> )</p> <p>(3)</p> <p>After that, the probability that a white ball is drawn from Q to R in the 2nd draw = <math>\frac{1}{5}</math></p> <p><math>\therefore</math> the probability that all balls in R are</p> <p>white = <math>\frac{1}{2} \times \frac{1}{5}</math> 1+1</p> <p><math>= \frac{1}{10}</math> .....</p> <p>0.1 <math>\frac{1}{6} \times \frac{1}{5}</math> <math>\frac{3}{30}</math></p> <p><math>1 - (\frac{3}{6} \times \frac{1}{5} + \frac{3}{6} \times \frac{4}{5})</math></p>	1A  1A	若不表示所求 概率者则 (PP-1)  若能表示理由 而有不正确答案 给全部分
	<u>1A</u> <u>3</u>	

SOLUTION		MARKS	REMARKS								
(Syllabus A)											
14. (a) (i)	$x^3 - 6x^2 + 9x - 1 = 0$										
(3)	$x^3 - 6x^2 + 9x = 1$	1M									
	Drawing the line $y = 1$ , the roots of the given equation were found to be 0.1, 2.3 and 3.5 (correct to 1 d.p.).	1A+1A	1 mark for 2 correct answers								
(ii)	$x^3 - 6x^2 + 10x - 6 = 0$										
(3)	$x^3 - 6x^2 + 9x = 6 - x$ .....	1M	for correct L.S.								
	Drawing the line $y = 6 - x$ , the root was found to be 3.8 (correct to 1 d.p.)	1A	for graph, $\pm$ /unit at (3,3), (4,2)								
		1A									
		<u>6</u>									
(b)											
(3)	<table border="1"><thead><tr><th>x</th><th><math>x^3 - 6x^2 + 10x - 6</math></th></tr></thead><tbody><tr><td>3.76</td><td>- (= -0.068)</td></tr><tr><td>3.77</td><td>+ (= 0.005)</td></tr><tr><td>3.765</td><td>- (= -0.031)</td></tr></tbody></table>	x	$x^3 - 6x^2 + 10x - 6$	3.76	- (= -0.068)	3.77	+ (= 0.005)	3.765	- (= -0.031)	1M	Change of sign, -ve for 3.765-3.769
x	$x^3 - 6x^2 + 10x - 6$										
3.76	- (= -0.068)										
3.77	+ (= 0.005)										
3.765	- (= -0.031)										
		1A	May use graphical method								
	$\therefore x = 3.77$ (correct to 2 d.p.)	1A									
		<u>3</u>									
(c)	Consider $x^3 - 6x^2 + 9x = k$	1M									
	From the graph, if $0 < k < 4$ ,	1A+1A	-1 for ' $<$ ' if otherwise correct.								
	the line $y = k$ meets the curve $y = x^3 - 6x^2 + 9x$ at three distinct points.		may omit								
	$\therefore x^3 - 6x^2 + 9x - k = 0$ has three distinct roots.	<u>3</u>									



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SOLUTION	MARKS	REMARKS
(Syllabus B)		
14. (a) Since $y \propto x$ and $z \propto \frac{1}{x}$ , $y = k_1x$ and $z = \frac{k_2}{x}$ (for some real $k_1, k_2$ ). $\therefore p = k_1x + \frac{k_2}{x}$	1A+1A	Accept $y = kx, z = \frac{k}{x}$
Putting $x = 2, p = 7$ , (or $x = 3, p = 8$ )	1M	
$7 = 2k_1 + \frac{k_2}{2}$	1A	
i.e. $4k_1 + k_2 = 14$		
Putting $x = 3, p = 8$ .		
$8 = 3k_1 + \frac{k_2}{3}$ .....	1A	
or $9k_1 + k_2 = 24$		
Solving these two equations,		
$5k_1 = 10$		
$k_1 = 2$	1A	
$k_2 = 6$	1A	
$\therefore p = 2x + \frac{6}{x}$		
When $x = 4$ , $p = 2(4) + \frac{6}{4}$ $= \frac{19}{2}$ .....	$\frac{1A}{8}$	
(b) $2x + \frac{6}{x} < 13$	1M	
$2x^2 - 13x + 6 < 0$ (as $x > 0$ )	1A	
$(2x - 1)(x - 6) < 0$		
$\therefore \frac{1}{2} < x < 6$	$\frac{2A}{4}$	-1 for ' $\leq$ '